Mathematics of Growth and Breakdown in the Bauer-Grossman Model:
All Value-Form, No Value-Substance

For 1st Grossman Class, September 24, 2023

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I. Symbols

<u>Value Magnitudes</u>

C (circulating) constant capital
S surplus-value
V variable capital
W total price (or value) of the product

<u>Physical Quantities</u>

A means of production
X total output produced

<u>Other</u>

$\bar{p}$ *constant* price (or value) of the commodity, per unit
r rate of profit
t time (the year)—used as subscript and as exponent. Note: the initial year here is $t = 0$.

II. Growth of Value Magnitudes

Bauer and Grossman assume that C grows at 10% per year, while V and S both grow at 5% per year. So

$$C_t = C_0(1.1)^t \quad (1)$$
$$V_t = V_0(1.05)^t \quad (2)$$
$$S_t = S_0(1.05)^t \quad (3)$$

III. Fall in Rate of Profit

The rate of profit, r, equals $S/(C + V)$. Using equations (1) through (3), we get:

$$r_t = \frac{S_0(1.05)^t}{C_0(1.1)^t + V_0(1.05)^t} = \frac{S_0\left(\frac{1.05}{1.1}\right)^t}{C_0 + V_0\left(\frac{1.05}{1.1}\right)^t} \quad (4)$$

$\frac{1.05}{1.1} < 1$, so $\left(\frac{1.05}{1.1}\right)^t$ decays over time (i.e, as the value of t increases); it approaches 0. And thus the numerator of the rate of profit approaches 0, while its denominator approaches C_0, which is positive. The rate of profit therefore approaches 0.

Marxist terminology about value magnitudes and relations between them notwithstanding, we will see that the fall in the rate of profit in the Bauer-Grossman model is *physically* determined. It's all value-form, no value-substance.

IV. Determinants of the Value Magnitudes

A. Expressing V_t and S_t in terms of other variables

This subsection is tedious, but we will need the end result, equation (11), for other things.

According to Marx's theory,

$$W_t = C_t + V_t + S_t \quad (5)$$

and we know, from (2) and (3) above, that

$$V_t + S_t = (V_0 + S_0)(1.05)^t \quad (6)$$

Equation (5) also implies that, at time $t = 0$,

$$W_0 = C_o + V_0 + S_0, \quad (7)$$

so that

$$V_0 + S_0 = W_0 - C_o \quad (8)$$

W_0 equals the (constant) price of the commodity times year 0's total physical output:

$$W_0 = \bar{p}X_0 \quad (9)$$

and C_0 equals the (constant) price of the commodity times the physical amount of the commodity that is employed as the means of production (and used up) in year 0:

$$C_0 = \bar{p}A_0 \quad (10)$$

We can therefore rewrite (6), using (8), (9), and (10), as

$$V_t + S_t = \bar{p}(X_0 - A_0)(1.05)^t \quad (11)$$

B. Determination of C_t

We know from equation (1) that $C_t = C_0(1.1)^t$ and, from (10), that $C_0 = \bar{p}A_0$. So

$$C_t = \bar{p}A_0(1.1)^t \quad (12)$$

It is also the case that C_t equals the price of the commodity times the physical amount of means of production:

$$C_t = \bar{p}A_t \quad (13)$$

and it follows from (12) and (13) that

$$A_t = A_0(1.1)^t \quad (14)$$

C. Determination of W_t

Since $W_t = C_t + V_t + S_t$ (eq. 5), we can use (11) and (12) to express W_t as

$$W_t = \bar{p}A_0(1.1)^t + \bar{p}(X_0 - A_0)(1.05)^t \tag{15}$$

W_t is also equal to the price of the commodity times the amount of physical output produced:

$$W_t = \bar{p}X_t \tag{16}$$

V. Implicit Constraint on Physical Output

The Bauer-Grossman scheme imposes a strict, though implicit, constraint on the production of physical output, *X.* Taken together, equations (15) and (16) tell us that:

$$\bar{p}X_t = \bar{p}A_0(1.1)^t + \bar{p}(X_0 - A_0)(1.05)^t \tag{17}$$

and after dividing everything in (17) by $\bar{p}$, we get the growth path of physical output,

$$X_t = A_0(1.1)^t + (X_0 - A_0)(1.05)^t \tag{18}$$

This implies that *the growth rate of physical output must always be less than the growth rate of physical means of production* (given in eq. 14). The latter grows by 10% per year, but the former grows by a smaller percentage, since only its first right-hand side term grows by 10%, while the other term grows by only 5%.

As we will see, the scheme's falling rate of profit and its breakdown tendency have everything to do this constraint on the growth of physical output.

VI. The Ever-Rising Constant-Capital Share of Total Value

In Grossman's numerical tables, the physical constraint appears only implicitly: (circulating) constant capital grows more rapidly than the total value of the product, and thus the ratio of these magnitudes rises continually throughout time. However, this ratio of two *value* magnitudes is identical to, and determined by, the ratio of *physical* means of production to *physical* output, i.e., the so-called "capital/output" ratio. Using equations (13) and (16), we see that

$$\frac{C_t}{W_t} = \frac{\bar{p}A_t}{\bar{p}X_t} = \frac{A_t}{X_t} \tag{19}$$

Now, equations (14) and (18) tell us that

$$\frac{A_t}{X_t} = \frac{A_0(1.1)^t}{A_0(1.1)^t + (X_0 - A_0)(1.05)^t} \tag{20}$$

Or, after dividing both the numerator and the denominator of the right-hand side by $A_0(1.1)^t$,

$$\frac{A_t}{X_t} = \frac{1}{1 + \frac{(X_0 - A_0)}{A_0}\left(\frac{1.05}{1.1}\right)^t} \tag{21}$$

Because $X_0 - A_0 > 0$, the denominator of (21) is always larger than the numerator, and thus the "capital/output" ratio is always less than 1. But because $\frac{1.05}{1.1}$ is less than 1, $\left(\frac{1.05}{1.1}\right)^t$ decays as time proceeds (i.e., as the value of t increases), and thus the denominator of (21) decreases and approaches 1 over time. The "capital/output" ratio itself therefore also approaches 1 as t increases. If the commodity produced is corn, this means that, eventually, almost one full bushel of seed corn is needed to produce each bushel of corn. It is hard to see how this kind of technical change can be called "technological progress." (For further discussion, see the end of section 3 of my Oct. 7, 2021 article on Grossman's theory.)

VII. The Falling Rate of Profit Reconsidered

Since the rate of profit, r, equals $S/(C+V)$, and we know from eq. (5) that $W = C + V + S$,

$$r = \frac{C+V+S}{C+V} - 1 = \frac{W}{C+V} - 1 \tag{22}$$

The smaller V is, the greater the rate of profit is. If V were at its minimum, $V = 0$, the rate of profit would be at its maximum. So the maximum rate of profit is

$$\max r = \frac{W}{C} - 1 \tag{23}$$

$\frac{W}{C}$ is the reciprocal of the constant-capital share of total value, $\frac{C}{W}$. Because the commodity's value (or price) is constant, $\frac{C}{W}$ will always be equal to $\frac{A}{X}$, the physical "capital/output" ratio (see eq. 19). The maximum rate of profit will therefore always be equal to the reciprocal of $\frac{A}{X}$, and so, using (21), we can express the maximum rate of profit as

$$\max r = \left\{1 + \frac{(X_0 - A_0)}{A_0}\left(\frac{1.05}{1.1}\right)^t\right\} - 1 = \frac{(X_0 - A_0)}{A_0}\left(\frac{1.05}{1.1}\right)^t \tag{24}$$

Because $\left(\frac{1.05}{1.1}\right)^t$ decays as time goes on (i.e., as t increases), the maximum rate of profit approaches 0, and the actual rate of profit cannot be bigger than the maximum rate. The cause of this fall, as we see, is the ever-rising physical "capital/output" ratio. It has nothing to do with Marx's value theory. The falling rate of profit of the Bauer-Grossman model is all value-form, no value-substance.

VIII. Grossman's Breakdown Condition

Grossman expressed his breakdown condition as follows. There is not enough surplus-value to continue capital accumulation at the pace that the model assumes:

$$S_t < (C_{t+1} - C_t) + (V_{t+1} - V_t) \tag{25}$$

Adding $C_t + V_t$ to both sides of (25), we obtain

$$C_t + V_t + S_t < C_{t+1} + V_{t+1} \tag{26}$$

or, using (5),

$$W_t < C_{t+1} + V_{t+1} \tag{27}$$

Thus, if W_t is less than C_{t+1} *alone*, there certainly must be a breakdown. In the Bauer-Grossman model, W_t must indeed eventually be less than C_{t+1}. Using (12) and (15), we see that

$$W_t < C_{t+1} \tag{28}$$

implies that

$$\bar{p}A_0(1.1)^t + \bar{p}(X_0 - A_0)(1.05)^t < \bar{p}A_0(1.1)^{t+1} \tag{29}$$

$$A_0(1.1)^t + (X_0 - A_0)(1.05)^t < A_0(1.1)^{t+1} \tag{30}$$

$$(X_0 - A_0)(1.05)^t < A_0(1.1)^{t+1} - A_0(1.1)^t \tag{31}$$

$$(X_0 - A_0)(1.05)^t < (1.1 - 1)A_0(1.1)^t \tag{32}$$

$$(X_0 - A_0)\left(\frac{1.05}{1.1}\right)^t < (0.1)A_0 \tag{33}$$

Because $\left(\frac{1.05}{1.1}\right)^t$ decays as *t* increases, inequality (33) must eventually hold true, and $W_t < C_{t+1}$ must therefore eventually hold true as well.

But why does it hold true? We know, from (13) and (16), that $W_t < C_{t+1}$ implies that

$$\bar{p}X_t < \bar{p}A_{t+1} \tag{34}$$

and therefore implies that

$$X_t < A_{t+1} \tag{35}$$

It is thus clear that the cause of breakdown is the *constraint on the growth of physical output*. The current year's physical output must eventually be less than the amount of means of production that is needed next year, given the assumed rate of capital accumulation.