

# Offstage, Directing the Action

## The Physical Dimension of Grossman's Breakdown Scheme

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We have seen that the basis of *value* is the fact that human beings relate to each other's labour as equal .... This is an abstraction, like all human thought, and social relations only exist among human beings to the extent that they think, and possess this power of abstraction from sensuous individuality and contingency. The kind of political economist who ... [insists] that the work performed by 2 individuals during the same time is not *absolutely equal* ..., doesn't yet even know what distinguishes human social relations from relations between animals. He is a beast.

—[Karl Marx](#), Economic Manuscript of 1861-63

Offstage, I couldn't put things into words, and that was the one thing I'd always been able to rely on.

—Carrie Fisher, [Shockaholic](#), p. 17

It's alright if you love me  
It's alright if you don't ...  
Breakdown, it's alright

—Tom Petty and the Heartbreakers, [Breakdown](#)

Henryk Grossman and his followers have continually claimed that his breakdown theory makes explicit and further develops a theory of capitalist economic breakdown that was implicit in Marx's *Capital*, but not fully worked out. I recently wrote an academic paper, "Grossman's Breakdown Theory versus Marx's Value Theory" (Kliman 2025), which showed that it just ain't so; the two theories are incompatible. The paper will appear in a forthcoming special issue of [Research in Political Economy](#).

I sent a draft version of it to several prominent followers of Grossman. Only one responded to the substance; and he responded only to part of it. To protect his anonymity, I will refer to him as Simplicio (a conversant in Galileo's [famous 1632 book](#) and [less-famous 1638 book](#)).

The present article is my reply to Simplicio. In the first section, I summarize what my forthcoming paper argues and demonstrates. In the second section, I discuss and reply to Simplicio's two main claims: (a) contrary to what my paper argues, there are no physical (use-value) relations implicit in Grossman's scheme of reproduction, and (b) contrary to my claim that Grossman's breakdown tendency and associated phenomena are physically determined, analysis of the physical relations in his scheme's economy is impossible, since "use-values are incommensurable" and new kinds of products continually replace older ones. The third section—which is by far the longest, and heavily mathematical—also deals with claim (b). Building on the results demonstrated in my forthcoming paper regarding the physicalist character of Grossman's breakdown theory, I show in the third section that these results hold true even when there are heterogeneous ("incommensurable"), discontinued, and new products.

## 1. Summary of My Forthcoming Paper

The paper puts forward, and demonstrates, two main theses. First, Marx's value theory implies that Grossman's breakdown tendency does not exist. According to Grossman, the ratio of constant capital to new value tends to rise *without limit*.<sup>1</sup> If not interrupted by the economic downturns it triggers, the operation of this tendency would eventually lead to total breakdown. Marx's theory also says that constant capital grows faster than new value, so that the ratio of constant capital to new value rises. However, there is no breakdown tendency, because (if new value grows over time, as Grossman assumed) the ratio of constant capital to new value cannot rise *without limit*.<sup>2</sup>

Somewhat surprisingly, Simplicio had nothing to say about this demonstration, even though he sent me more than two dozen messages over a three-week period. I suspect that there are two reasons he failed to address this issue. One is that he was unable to challenge my demonstration. The other is that he is apparently not very interested in Marx's actual value theory or in whether Grossman's theory is compatible with it. This is something that I have observed among other followers of Grossman over the years. When they refer to "Marx" and "Marx's theory," what they actually mean is Grossman's theory and Grossman's take on Marx's theory.<sup>3</sup> These are the things that interest them, not Marx's theory itself.

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<sup>1</sup> The new value, which is added by living labor, is the sum of variable capital and surplus-value.

<sup>2</sup> In Grossman's reproduction scheme, the price of means of production is constant, so constant capital-value permanently grows at the same rate as means of production. Since this growth rate exceeds the growth rate of new value, the ratio of constant capital to new value rises without limit. But productivity is rising, so, according to Marx's theory, the price of means of production falls. This causes constant capital-value to grow more slowly than means of production and, ultimately, to grow at the same rate as new value. Hence, the ratio of constant capital to new value does not rise without limit.

<sup>3</sup> In a close examination of a few pages of Grossman's book, [Michael Rednitz has recently shown](#) that "Grossman's breakdown theory rests on a revision of Marx's original text. ... When correcting the 'distortions' resulting from Engels's editing error, or Marx's presumed writing error, it is actually Grossman who distorts Marx's reasoning; and Grossman necessarily depends on this distortion when defining his breakdown criterion."

All of Simplicio's responses pertained to the other thesis demonstrated in my paper: contrary to what Grossman and his followers have consistently claimed, his breakdown tendency and associated phenomena have nothing to do with value or value theory. In his reproduction scheme, the variables have the names of value magnitudes ("surplus-value," etc.), but this is all value-form, no value substance. The variables' movements are determined exclusively by movements in *physical quantities* (of products, means of production, etc.).

Specifically:

- (1) Grossman's breakdown condition—breakdown occurs when the amount of surplus-value actually created is less than the amount of surplus-value needed to fund capital accumulation, given the current growth rates of constant and variable capital—always reduces to a *purely physical* condition: breakdown occurs when the amount of physical output actually produced is less than the amount of output needed to satisfy the current demand for physical means of production and subsistence ("wage goods").
- (2) In the scheme of reproduction that Grossman took over from Otto Bauer, there is an implicit but strict constraint on the production of physical output: it cannot grow as rapidly as physical means of production. This leads to a disappearing net product (the difference between the total physical product and the means of production used up to produce it); over time, the net product becomes a vanishingly small share of the total physical product.

Consequently, in the Bauer-Grossman scheme,

- (2a) breakdown occurs because the amount of physical output that is produced eventually falls short of the amount needed to satisfy the demand for physical means of production and subsistence.

and

- (2b) the rate of profit continually falls, and approaches zero, because the net product becomes a vanishingly small share of the total physical product.

## 2. Simplicio's Response

### A. The "No Use-Values" Line

There were two main ways in which Simplicio responded to all this. One was to deny that there are any physical relations implicit in the Bauer-Grossman reproduction scheme: "Grossmann's model does not involve physical quantities." "[U]se-values are excluded from the analysis, which regards values alone."

This is sheer lunacy. Physical quantities of output, means of production, and so on are not *depicted* in Grossman's numerical tables, but they are *present* throughout his analysis. They have

to be. Constant capital, variable capital, surplus-value, and so on are not free-floating sums of value disconnected from material reality. The total value of each of them is the sum of the prices of their components times the physical quantities of these components.

There are three variables here: total value, price, and physical quantity. If we know the growth rates of any two of them, we can find the growth rate of the remaining one.<sup>4</sup> The Bauer-Grossman scheme tells us what the growth rates of the total values are. And it assumes that the prices remain constant over time,<sup>5</sup> which means that their growth rates equal 0. This gives us sufficient information to find the growth rates of the physical quantities. So although they are not depicted in Grossman's numerical tables, the physical quantities are present, offstage. And it turns out that they direct all the action.

Grossman himself was acutely aware that physical quantities were present in his analysis of the reproduction process. As he stated in the introduction to his book,

the problem dealt with here is the central problem or rather *the* problem of capitalism ...[, ] whether fully developed capitalism ... is capable of developing the process of reproduction indefinitely and on a continually expanding basis ....

The specific nature of capitalist commodity production is apparent in the fact that it is not simply a *labour process* in which products are created by the elements of production *mp* and *L* [means of production and labor-power]. Rather, ... it is simultaneously a labour process for the creation of products and a *valorisation process*. The elements of production *mp* and *L* figure not only in this natural form, but simultaneously also as values *c* and *v* [constant capital and variable capital] .... [T]he expansion of *mp* in relation to *L* occurs on the basis of the law of value. [Grossman 2022, pp. 50-51, emphases in original]

This passage, just like the Bauer-Grossman reproduction scheme, is about “the process of reproduction.” In this one brief passage, Grossman refers to commodity production, to products (twice), to means of production (three times), and to labor-power (three times). Simplicio would have us believe, to the contrary, that Grossman presented and analyzed a system of capitalist commodity production in which there are no commodities, by means of a scheme of reproduction in which no use-values are reproduced!

Furthermore, Grossman (2022, p. 124, p. 124 n68, p. 125, p. 129, emphases added) noted that the reproduction scheme he took over from Bauer “takes account of incessant *technological advances*, i.e. the development of the productive forces”; that the additional capital invested each year is “expended not just on wages but also on *means of production, such as machinery, raw materials, etc.*”; that “growth in *productivity*” is a feature of the scheme; and that “we have constant capital growing at 10 per cent a year, which expresses *technological progress*, twice as fast as the annual increase in population.” Thus, even though the numerical tables depict only accumulation of sums of value, what the changes in these sums of value reflect are changing

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<sup>4</sup> If  $A = B \times C$ , then the growth rate of  $A$  equals the growth rate of  $B$  plus the growth rate of  $C$  plus the product of the growth rates of  $B$  and  $C$ .

<sup>5</sup> Simplicio denied this, but, as I will discuss below, the evidence is incontrovertible.

physical relations that are also present, though offstage. The additional constant capital purchases additional physical means of production. There is incessant adoption of new technologies that require relatively more means of production and relatively less living labor. And this technological progress leads to increases in productivity—that is, in physical output per unit of labor.

I pointed out to Simplicio that growth of physical means of production is implicit in the scheme's numerical tables, and that Grossman himself commented on this fact. He wrote:

In Table 2 (on page 136) we saw that, assuming population growth is five per cent a year and constant capital increases by 10 per cent a year, the capitalist mechanism described must collapse in year 35. As has been demonstrated here, however, the mass of capital grows more rapidly in use values than in value terms .... *That means, however, that the breakdown tendency is weakened.* The breakdown will not occur in year 35, as shown in the table (that is when only its value aspect alone is considered) but at a *later* point, perhaps in year 40 or 45. [Grossman 2022, p. 297, emphases in original]

Grossman was saying, quite obviously, that breakdown occurs in year 35 of his numerical scheme (Table 2) because the scheme assumes that physical means of production (“the mass of capital ... in use values”) grow only as rapidly as constant capital (“in value terms”). In fact, however, physical means of production grow more rapidly than constant capital-value, so the breakdown will occur at a later point.

Simplicio's interpretation of the passage was quite different: when Grossman indicated that the scheme's means of production do not grow *more rapidly*, what he meant is that they do not grow *at all*, because “use-values are excluded” from the scheme. “Since there are no use-values, they can't grow.”

Once again, sheer lunacy. Nothing in the passage suggests that the scheme's means of production do not grow at all. Simplicio pulled this out of the hat, and his interpretation is very strained. The natural way to read “however, ... grows more rapidly” is “however, ... grows more rapidly, not only as rapidly” or, possibly, “however, ... grows more rapidly, not less rapidly,” but not “however, ... grows more rapidly rather than not at all.”

In addition, Simplicio's argument makes absolutely no economic sense. Constant capital is the amount of value *invested in means of production*. If there are no means of production, there is no constant capital, much less a constant capital that grows by 10% annually. “Since there are no use-values, there are no values, so they can't grow” (see the “no use-values” version of Grossman's Table 2 on the next page).

Tabula Rasa: The “No Use-Values” Version of Grossman’s Table 2

Tab. II Die Fortsetzung des Bauerschen Reproduktionsschemas

	c	v	k	$a_c$	$a_v$	Jährlicher Produkten- wert	Konsum der Kapitalisten k in % des Mehrerts	Die Akku- mulationsrate $a_c + a_v$ in % des Mehrerts	Profitrate: $\frac{k + a_c + a_v}{c + v}$
5. Jahr	+	+	+	+	+	=			
6. Jahr	+	+	+	+	+	=			
7. Jahr	+	+	+	+	+	=			
8. Jahr	+	+	+	+	+	=			
9. Jahr	+	+	+	+	+	=			
10. Jahr	+	+	+	+	+	=			
11. Jahr	+	+	+	+	+	=			
15. Jahr	+	+	+	+	+	=			
19. Jahr	+	+	+	+	+	=			
20. Jahr	+	+	+	+	+	=			
21. Jahr	+	+	+	+	+	=			
25. Jahr	+	+	+	+	+	=			
27. Jahr	+	+	+	+	+	=			
30. Jahr	+	+	+	+	+	=			
31. Jahr	+	+	+	+	+	=			
33. Jahr	+	+	+	+	+	=			
34. Jahr	+	+	+	+	+	=			
35. Jahr	+	+	+	+	+	=			

## B. The Multiple, Ever-Changing, Incommensurable Use-Values Line

Simplicio’s other main response to the points demonstrated in my paper was that it is impossible to analyze the physical relations implicit in the Bauer-Grossman scheme. He rejected my one-sector analysis of the scheme (in which there is one produced commodity and thus one price) on the grounds that Grossman was considering a multisector economy. Moreover, he insisted that Grossman’s scheme depicts an economy in which physical products and means of production are continually being discontinued, and replaced by different ones, as a result of incessant technological change. Analysis of the long-run growth paths of physical variables is therefore impossible, because these variables do not have long-run growth paths.

Notice that this response *completely contradicts* Simplicio’s other—“use-values are excluded,” “there are no use-values”—line of defense.

In reply to his objection to my one-sector analysis, I pointed out that such an analysis is equivalent to the analysis of the aggregate magnitudes of a multisector economy: “It is equally correct ... to think of the price as the aggregate price *index*, and to think of the physical quantities of output and means of production as quantity *indexes* of aggregate output and the aggregate

stock of means of production (i.e., real output and real means of production).”<sup>6</sup> Use of aggregate quantity and price indexes is a mainstay of macroeconomics. For example, consider the well-known Gross Domestic Product (GDP) measure. Current-dollar GDP is a monetary measure, but the far more important “real GDP” or constant-dollar GDP is a quantity index, a single number that measures the level of physical production of all “final” goods and services that are newly produced within a country. Associated with it is the GDP price index, which similarly represents the prices of all these goods and services in the form of a single number.<sup>7</sup>

Simplicio did not respond to this point.

I also pointed out that my electronic search of Grossman’s (2022) book returned nine hits for “price level”—a technical term that basically means the same thing as “price index”—as well as Figure 4 on page 277 (see screenshot below).

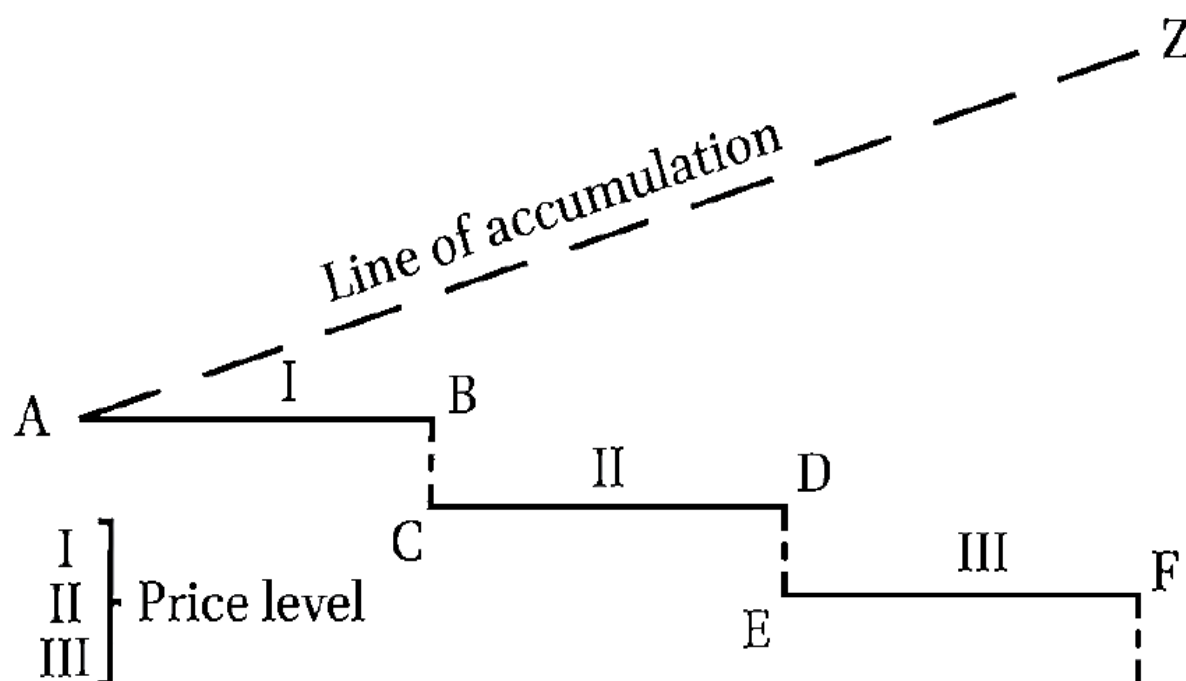


FIGURE 4

<sup>6</sup> Emphases in original. The quoted sentence also appears in the final version of my forthcoming paper.

<sup>7</sup> Every quantity index has an associated price index and vice-versa. The total value of the monetary aggregate (e.g., current-dollar GDP) is the product of the quantity and price indexes.

Simplicio did not respond to my point about Grossman's use of the concept of price level.<sup>8</sup> He persisted in his beastly resistance to the "power of abstraction from sensuous individuality and contingency" (see this article's first epigraph), insisting that "use-values are incommensurable" and therefore that the physical relations of a multisector economy cannot be analyzed by means of a one-sector or aggregative analysis. "[O]ne can't sum over [i.e., add together] spindles, rubber, and bulldozers to measure a growth rate of means of production."

*In light of this resistance to the power of abstraction, the remainder of the present article will respond to Simplicio's objections by showing that the points demonstrated in my forthcoming paper (summarized in points (1), (2), (2a), and (2b) above) hold true in a multisector context, including in situations in which some products and means of production are discontinued and/or new. The analysis will employ mathematically valid operations, not "sum over spindles, rubber, and bulldozers." It will take into account all of the myriad products and means of production, and it will respect the fact that they and their prices are all different from one another.*

### C. Just-So Storytelling

Before turning to that analysis, a final comment about Simplicio's second line of response is in order. I was struck by how weak and indecisive it is. He was not actually *denying* my claim that Grossman's breakdown tendency and associated phenomena are driven by movements in physical quantities. He was *pleading ignorance*, alleging that the physical dimension of the scheme's economy simply cannot be analyzed.

If that were true, then the value-theoretic explanation of the breakdown tendency and associated phenomena—the explanation that Grossman and his many followers have provided—might be right, but it might be wrong. My contrary explanation might be right or wrong. And there would be no way to distinguish what's right from what's wrong.

For example, Simplicio commented at one point that Grossman's "position is that value relations—in particular the rate and mass of profit—determine the evolution of the capitalist system." Fine, value relations determine the evolution of the capitalist system. *But what determines the value relations?!* The rate and mass of profit are not prime movers that descend from heaven and set everything else in motion. Changes in their magnitudes are determined by other events and processes. What we want to know, or should want to know, is what these other events and processes are and how they cause the rate and mass of profit to change.

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<sup>8</sup> Simplicio did deny that prices are held constant in the Bauer-Grossman scheme: "the 'constancy of prices' does not refer to the values of goods ... but to the assumption that prices = values. ... There is no premise that values are constant, only that prices = values." Why, then, is there a flat price level between points A and B in Grossman's Figure 4, rather than an upward- or downward-sloping price level? And Grossman (2022, p. 319, emphasis in original) stated categorically that "the assumption of *constant value* is one of many underlying Marx's reproduction schema. Accordingly, Bauer makes the constancy of values a basic part of his reproduction schema." *Didn't Grossman mean by this that Bauer makes the constancy of values a basic part of his reproduction schema?!*



Specifically, are the falling rate of profit and the breakdown tendency *of the Bauer-Grossman scheme* physically determined, as I maintain, or are they not? Yes, yes, I know—they are determined by “the rising organic composition of capital.” But that answer just begs the question. Is the “rising organic composition of capital” *of the Bauer-Grossman scheme* physically determined, as I maintain, or is it not? When Simplicio said that the physical dimension of the scheme’s economy cannot be analyzed, he was saying, in effect, that he doesn’t know the answer and that none of us can know the answer.

But if that were the case, what would entitle him to keep repeating Grossman’s value-theoretic interpretation of his breakdown tendency and associated phenomena, rather than my contrary interpretation? What would entitle him to apply Grossman’s interpretation to the analysis of current economic phenomena? Nothing at all. If Simplicio were to continue down that path, he would simply be telling us just-so stories—making claims, and trying to persuade others to accept claims, that he doesn’t and can’t know to be true.

Fortunately, however, there *is* a way to distinguish right from wrong here. The physical dimension of the Bauer-Grossman scheme *can* indeed be analyzed, even when there are multiple, discontinued, and new use-values.

### 3. Grossman’s Physically-Determined Breakdown Theory, in a Multisector Context, with Discontinued and New Products

#### A. Notation

My analysis will employ the following symbols:

##### Value Magnitudes

$C$	(consumed) constant capital
$S$	surplus-value
$V$	variable capital
$W$	total value (or price) of the product

##### Physical Quantities

$A$	means of production
$B$	means of subsistence (“wage goods” consumed by workers)
$K$	demand for means of production and subsistence; $K = A + B$
$X$	total output produced

##### Growth Rates

$g^A$	growth rate of means of production
$g^C$	growth rate of constant capital
$g^L$	growth rate of employed labor force, variable capital, and surplus-value

### Weights

$\omega$	share of total expenditures on output
$\acute{\omega}$	share of total expenditures on means of production

### Other

$p$	price of a commodity, per unit
$r$	rate of profit
$t$	time (used as a subscript and as an exponent)

Note that the time subscript  $t$  will refer to a *moment* in time, not to a period of time. Grossman's Year 1 starts at time  $t = 0$  and ends at time  $t = 1$ , his Year 2 starts at time  $t = 1$  and ends at time  $t = 2$ , and so on. To reflect the fact that the Bauer-Grossman scheme of reproduction conceives of inputs as entering into production at the start of the year and output as emerging at the end, the initial values of  $C$ ,  $V$ ,  $A$ ,  $B$ , and  $K$  are denoted as  $C_0$ ,  $V_0$ ,  $A_0$ ,  $B_0$ , and  $K_0$ , while the initial values of  $W$  and  $X$  are denoted as  $W_1$  and  $X_1$ . Since surplus-value is created throughout the year, the initial values of  $S$  and  $r$  are denoted as  $S_{0,1}$  and  $r_{0,1}$ . Similarly, variables grow from one year to the next, so the initial values of the growth rates are denoted as  $g_{0,1}^A$ ,  $g_{0,1}^C$ , and  $g_{0,1}^L$ .

In addition to the symbols listed above,  $j$ ,  $n$ , and  $\sum$  will be used in connection with summation notation. All summations will be of the form  $\sum_{j=1}^n Z_{jt}$  (although, to reduce clutter, I will write this from now on as  $\sum Z_{jt}$ ). Here,  $Z$  is the variable whose numerical values are being added together,  $\sum$  (for "sum" or "summation") indicates that a series of terms is being added together, and the subscript  $j = 1$  and superscript  $n$  indicate that the value of each  $Z_{jt}$ , starting at  $Z_{1t}$  and stopping at  $Z_{nt}$ , is included in the series. (Thus,  $j$  takes on, in succession, the values 1, 2, ...,  $n - 1$ ,  $n$ ). In other words,  $\sum_{j=1}^n Z_{jt} = Z_{1t} + Z_{2t} + \cdots + Z_{(n-1)t} + Z_{nt}$ .<sup>9</sup>

The analysis will be all-inclusive; by summing from 1 to  $n$ , we take the whole economy into account. This implies that an unspecified number,  $n$ , of different kinds of products is produced. But the number of products produced can vary over time, so the value of  $n$  at time  $t+1$  need not equal the value of  $n$  at time  $t$ .

Some or all of the products serve as means of production and/or means of subsistence. There can be some that serve as neither. For example, at time  $t$ ,  $X_{3t}$  is the total amount of product 3 that is produced, and  $K_{3t}$  and  $\frac{K_{3t}}{X_{3t}}$  are, respectively, the amount of and the share of product 3 that are demanded as a means of production and/or means of subsistence. If none of product 3 is demanded for either purpose, then  $K_{3t+1} = \frac{K_{3t+1}}{X_{3t+1}} = 0$ .

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<sup>9</sup> For further explanation of summation notation, see <https://www.columbia.edu/itc/sipa/math/summation.html>.

At time  $t$ , the aggregate amounts of constant capital, variable capital, surplus-value, and the value of output are  $\sum C_{jt}$ ,  $\sum V_{jt}$ ,  $\sum S_{jt}$ , and  $\sum W_{jt}$ , respectively. To reduce clutter, I will refer to these sums as  $C_t$ ,  $V_t$ ,  $S_t$ , and  $W_t$ .

## B. Grossman's Breakdown Condition

The breakdown point in Grossman's theory is the point in time at which the amount of surplus-value that has been created becomes less than the amount of surplus-value that is needed to fund accumulation, given the current rates of growth of constant and variable capital. The breakdown condition is thus

$$S_{t,t+1} < (C_{t+1} - C_t) + (V_{t+1} - V_t) \quad (1)$$

where the differences on the right-hand side are the amounts of constant and variable capital that are newly accumulated. Adding  $C_t + V_t$  to both sides of Inequality (1), we obtain

$$C_t + V_t + S_{t,t+1} < C_t + V_t + (C_{t+1} - C_t) + (V_{t+1} - V_t) \quad (2)$$

or, since the total value of output,  $W_{t+1}$ , equals the sum of constant capital, variable capital, and surplus-value,

$$W_{t+1} < C_{t+1} + V_{t+1} \quad (3)$$

which implies that

$$\frac{C_{t+1} + V_{t+1}}{W_{t+1}} > 1 \quad (4)$$

But  $C_{t+1}$ ,  $V_{t+1}$ , and  $W_{t+1}$  are sums of components, and each component is a price times a physical quantity. Specifically,

$$C_t = \sum p_{jt} A_{jt} \quad (5)$$

$$V_t = \sum p_{jt} B_{jt} \quad (6)$$

$$C_{t+1} + V_{t+1} = \sum p_{jt+1} K_{jt+1} \quad (7)$$

$$W_{t+1} = \sum p_{jt+1} X_{jt+1} \quad (8)$$

We can therefore express the left-hand side of Inequality (4) in terms of weights and the shares of physical products that are accumulated as means of production and/or subsistence:

$$\frac{C_{t+1} + V_{t+1}}{W_{t+1}} = \frac{p_{1t+1}K_{1t+1} + \dots + p_{nt+1}K_{nt+1}}{\sum p_{jt+1}X_{jt+1}} = \frac{p_{1t+1}K_{1t+1}}{\sum p_{jt+1}X_{jt+1}} + \dots + \frac{p_{nt+1}K_{nt+1}}{\sum p_{jt+1}X_{jt+1}}$$

$$\begin{aligned}
&= \left( \frac{p_{1t+1}X_{1t+1}}{\sum p_{jt+1}X_{jt+1}} \right) \left( \frac{p_{1t+1}K_{1t+1}}{p_{1t+1}X_{1t+1}} \right) + \dots + \left( \frac{p_{nt+1}X_{nt+1}}{\sum p_{jt+1}X_{jt+1}} \right) \left( \frac{p_{nt+1}K_{nt+1}}{p_{nt+1}X_{nt+1}} \right) \\
&= \left( \frac{p_{1t+1}X_{1t+1}}{\sum p_{jt+1}X_{jt+1}} \right) \left( \frac{K_{1t+1}}{X_{1t+1}} \right) + \dots + \left( \frac{p_{nt+1}X_{nt+1}}{\sum p_{jt+1}X_{jt+1}} \right) \left( \frac{K_{nt+1}}{X_{nt+1}} \right)
\end{aligned} \tag{9}$$

Each of the weights, the  $\frac{p_{jt+1}X_{jt+1}}{\sum p_{jt+1}X_{jt+1}}$ , is the share of total expenditure, on all newly-produced products, that is spent on product  $j$ . If we relabel each weight as  $\omega_{jt+1}$ , the left-hand side of Inequality (4) can be expressed as

$$\omega_{1t+1} \left( \frac{K_{1t+1}}{X_{1t+1}} \right) + \dots + \omega_{nt+1} \left( \frac{K_{nt+1}}{X_{nt+1}} \right) = \sum \omega_{jt+1} \left( \frac{K_{jt+1}}{X_{jt+1}} \right) \tag{10}$$

and (4) can be restated as

$$\frac{C_{t+1} + V_{t+1}}{W_{t+1}} = \sum \omega_{jt+1} \left( \frac{K_{jt+1}}{X_{jt+1}} \right) > 1 \tag{11}$$

Each  $\frac{K_{jt+1}}{X_{jt+1}}$  is a “physical accumulation share,” the share of physical output  $j$  that is accumulated as physical means of production and physical means of subsistence for workers’ consumption. Inequality (11) says that Grossman’s breakdown condition implies that the weighted average of the “physical accumulation shares” exceeds 1. In other words, the economy breaks down when, on average, the physical amount of a product that needs to be accumulated exceeds the amount that has been produced.

At least one of the  $\frac{K_{jt+1}}{X_{jt+1}}$  terms must be greater than 1; in at least one case, in other words, the amount of the product that needs to be accumulated must exceed the amount of it that has been produced. To see this, note first that the weights sum to 1:

$$\sum \omega_{jt+1} = \sum \left( \frac{p_{jt+1}X_{jt+1}}{\sum p_{jt+1}X_{jt+1}} \right) = \frac{\sum p_{jt+1}X_{jt+1}}{\sum p_{jt+1}X_{jt+1}} = 1 \tag{12}$$

Inequality (11) can thus be restated, using Equation (12), as

$$\omega_{1t+1} \left( \frac{K_{1t+1}}{X_{1t+1}} \right) + \dots + \omega_{nt+1} \left( \frac{K_{nt+1}}{X_{nt+1}} \right) > \sum \omega_{jt+1} \tag{13}$$

or

$$\omega_{1t+1} \left( \frac{K_{1t+1}}{X_{1t+1}} - 1 \right) + \dots + \omega_{nt+1} \left( \frac{K_{nt+1}}{X_{nt+1}} - 1 \right) > 0 \tag{14}$$

But the left-hand side of Inequality (14) can exceed 0 only if at least one of the  $\frac{K_{jt+1}}{X_{jt+1}} - 1$  terms exceeds 0, which means that at least one of the  $\frac{K_{jt+1}}{X_{jt+1}}$  terms must exceed 1.

It is of course impossible that more of a newly-produced commodity is accumulated than the amount that has been produced. If impossibility admitted of degrees, it would be even more impossible that this holds true on average. So Grossman was absolutely right about the trajectory of the Bauer-Grossman scheme of reproduction: the economy must eventually break down.

The point, however, is that further economic expansion has become *physically* impossible. It has nothing to do with value or value theory. Although the breakdown condition superficially appears to be about an amount of *surplus-value* that is insufficient for continued capital accumulation at current growth rates, it is in fact about amounts of *physical output* that are insufficient. Not enough physical stuff has been produced to permit means of production and subsistence to accumulate at their current rates.

It is important to note that Grossman's breakdown condition *always* reduces to this physical impossibility. *This is true whether there is one commodity or multiple ones, whether prices remain constant or vary over time, and whether the same physical products are reproduced or new kinds of products replace old ones.* When deriving Inequality (11) from Grossman's own formulation of the breakdown condition (Inequality (1)), I allowed for multiple commodities; I did not say whether the prices ( $p_{jt+1}$ ) are equal or unequal to past and future prices; and I did not say whether the physical outputs and the amounts of them that are accumulated ( $X_{jt+1}$  and  $K_{jt+1}$ ) are the same kinds of products that were previously produced.

### C. The Breakdown of the Economy in the Bauer-Grossman Scheme

The Bauer-Grossman scheme assumes that  $C$  grows by a constant 10% per year, while  $V$  and  $S$  both grow by a constant 5% per year, which is also the growth rate of the employed labor force. Thus,  $g^C = 10\%$  and  $g^L = 5\%$ . But these specific numbers affect only the rapidity of changes in the variables, not the general dynamical properties of the scheme. To generalize the scheme, I will treat the growth rates as parameters; they can take on any constant values as long as  $0 < g^L < g^C$ . The growth paths of  $C$ ,  $V$ , and  $S$  are therefore

$$C_t = C_0(1 + g^C)^t \quad (15)$$

$$V_t = V_0(1 + g^L)^t \quad (16)$$

$$S_{t,t+1} = S_{0,1}(1 + g^L)^t \quad (17)$$

In accordance with Marx's theory, the total value (or price) of the product is equal to the sum of these three amounts of value:

$$W_{t+1} = C_t + V_t + S_{t,t+1} = C_0(1 + g^C)^t + (V_0 + S_{0,1})(1 + g^L)^t \quad (18)$$

Some share of the total value of output, of time  $t+1$ , is accumulated as constant and variable capital. Using Equations (15), (16), and (18), we can express that share as

$$\frac{C_{t+1}+V_{t+1}}{W_{t+1}} = \frac{C_0(1+g^C)^{t+1} + V_0(1+g^L)^{t+1}}{C_0(1+g^C)^t + (V_0+S_{0,1})(1+g^L)^t} = \frac{C_0(1+g^C) + V_0(1+g^L)\left(\frac{1+g^L}{1+g^C}\right)^t}{C_0 + (V_0+S_{0,1})\left(\frac{1+g^L}{1+g^C}\right)^t} \quad (19)$$

Since  $g^L < g^C$ ,  $\left(\frac{1+g^L}{1+g^C}\right)^t$  decays and approaches 0 over time (i.e., as  $t$  increases).  $\frac{C_{t+1}+V_{t+1}}{W_{t+1}}$  therefore approaches  $\frac{C_0(1+g^C)}{C_0} = 1 + g^C > 1$ . The breakdown condition (Inequality (4)) is therefore eventually satisfied. But as we saw when deriving Inequality (11), the reason  $\frac{C_{t+1}+V_{t+1}}{W_{t+1}} > 1$  is that, on average, the physical amount of a product that is accumulated

exceeds the amount that has been produced (i.e.,  $\sum \omega_{jt+1} \left(\frac{K_{jt+1}}{X_{jt+1}}\right) > 1$ ). The breakdown of the Bauer-Grossman scheme's economy is therefore physically determined. Eventually, not enough physical stuff is produced to satisfy the demand for physical means of production and subsistence. And, it bears repeating, I have shown this to be true even when there are multiple products, new products, and discontinued products.

#### D. The Disappearing Net Product of the Bauer-Grossman Scheme

The remaining points I will demonstrate pertain to the Bauer-Grossman reproduction scheme's disappearing net product and falling rate of profit. These demonstrations involve the use of growth rates. Thus, to show that the phenomena under investigation are physically determined—even when there are discontinued and new commodities—I need to show that the growth rates of the scheme's value magnitudes are physically determined. To avoid interrupting the flow of the argument, I show this in an appendix at the end of the article.

Using Equations (15) and (18), we can see that, in the Bauer-Grossman scheme, the constant-capital share of the total value of output is

$$\frac{C_t}{W_{t+1}} = \frac{C_0(1+g^C)^t}{C_0(1+g^C)^t + (V_0+S_{0,1})(1+g^L)^t} = \frac{1}{1 + \left(\frac{V_0+S_{0,1}}{C_0}\right)\left(\frac{1+g^L}{1+g^C}\right)^t} \quad (20)$$

Since  $g^L < g^C$ ,  $\left(\frac{1+g^L}{1+g^C}\right)^t$  decays and approaches 0 over time.  $\frac{C_t}{W_{t+1}}$  therefore increases continually and approaches 1. But why?

Well, we know, from Equations (1) and (4), that

$$\frac{C_t}{W_{t+1}} = \frac{\sum p_{jt}A_{jt}}{\sum p_{jt+1}X_{jt+1}} = \frac{p_{1t}A_{1t} + \dots + p_{nt}A_{nt}}{\sum p_{jt+1}X_{jt+1}} \quad (21)$$

and since prices are held constant in the Bauer-Grossman scheme, we have

$$\begin{aligned}
\frac{C_t}{W_{t+1}} &= \frac{p_1 A_{1t} + \dots + p_n A_{nt}}{\sum p_j X_{jt+1}} \\
&= \left( \frac{p_1 X_{1t+1}}{\sum p_j X_{jt+1}} \right) \left( \frac{p_1 A_{1t}}{p_1 X_{1t+1}} \right) + \dots + \left( \frac{p_n X_{nt+1}}{\sum p_j X_{jt+1}} \right) \left( \frac{p_n A_{nt}}{p_n X_{nt+1}} \right) \\
&= \left( \frac{p_1 X_{1t+1}}{\sum p_j X_{jt+1}} \right) \left( \frac{A_{1t}}{X_{1t+1}} \right) + \dots + \left( \frac{p_n X_{nt+1}}{\sum p_j X_{jt+1}} \right) \left( \frac{A_{nt}}{X_{nt+1}} \right) \\
&= \omega_{1t} \left( \frac{A_{1t}}{X_{1t+1}} \right) + \dots + \omega_{nt} \left( \frac{A_{nt}}{X_{nt+1}} \right) = \sum \omega_{jt} \left( \frac{A_{jt}}{X_{jt+1}} \right) \tag{22}
\end{aligned}$$

where  $\frac{A_{jt}}{X_{jt+1}}$  is the *physical* “capital-output ratio” of the  $j$ -th commodity, the amount of that commodity used up in production as a share of the amount of it newly produced.

Thus, in the Bauer-Grossman scheme,  $\frac{C_t}{W_{t+1}}$  is just the weighted average physical “capital-output ratio” in disguise. It increases continually and approaches 1 because *the weighted average of the physical “capital-output ratios” continually increases and approaches 1*. But this means that, on average, the physical net product disappears. That is, the weighted average difference between the amount of a product that is newly produced, and the amount of that product that is used up in production ( $X_{jt+1} - A_{jt}$ ), becomes a vanishingly small share of the total product.

However, there is one wrinkle we have to iron out. If a product is discontinued (i.e., if it serves as a means of production at time  $t$  but is not reproduced as an output at time  $t+1$ ), the total product ( $X_{jt+1}$ ) does not exist. So we cannot divide the total product into one part that replaces used-up means of production and another part that is net product. In the aggregate, however, the *value* of the total product ( $W_{t+1}$ ) continues to exist, and it can be divided into two parts, one that represents used-up means of production ( $C_t$ ) and another that represents the net product ( $W_{t+1} - C_t$ ). The question then becomes: Is the net-product share of the value of the total product determined exclusively by physical factors? I will now show that, in the Bauer-Grossman scheme, the answer is “yes.”

The net-product share of the total value is  $\frac{W_{t+1} - C_t}{W_{t+1}} = 1 - \frac{C_t}{W_{t+1}}$ . We already know that, in the Bauer-Grossman scheme,  $\frac{C_t}{W_{t+1}}$  rises continuously toward a limiting value of 1 (see Equation (20)), so that the net-product share of the total value declines continuously and approaches 0. The only remaining task is to show that this decline is physically determined, even when one or more products are discontinued.

Equations (9) through (11) showed that  $\frac{C_{t+1}+V_{t+1}}{W_{t+1}}$  is always equal to  $\sum \omega_{jt+1} \left( \frac{K_{jt+1}}{X_{jt+1}} \right)$   
 $= \sum \omega_{jt+1} \left( \frac{A_{jt+1}+B_{jt+1}}{X_{jt+1}} \right)$ . This holds true even if some products are discontinued between  
times  $t$  and  $t+1$ , since the equation pertains solely to those that are produced at time  $t+1$ . And  
since  $V_{t+1} = 0$  if all  $B_{jt+1} = 0$  (see Equation (6)), it follows that

$$\frac{C_{t+1}}{W_{t+1}} = \sum \omega_{jt+1} \left( \frac{A_{jt+1}}{X_{jt+1}} \right) \quad (23)$$

But  $C_{t+1} = (1 + g^C)C_t$ , so it follows from Equation (23) that

$$\frac{C_t}{W_{t+1}} = \frac{\sum \omega_{jt+1} \left( \frac{A_{jt+1}}{X_{jt+1}} \right)}{1+g^C} \cdot 10 \quad (24)$$

The numerator of the right-hand side of Equation (24) is the weighted average of the shares of  
physical output that are accumulated as means of production. In the appendix, I demonstrate that  
the denominator,  $1 + g^C$ , is also physically determined (see Equations (A5) and (A8)). Thus,  
even when some products are discontinued, the rise in  $\frac{C_t}{W_{t+1}}$  toward the limiting value of 1 is  
physically determined. The decline in the net-product share of the total value of output,  
 $\frac{W_{t+1}-C_t}{W_{t+1}} = 1 - \frac{C_t}{W_{t+1}}$ , is therefore physically determined as well.

To see this more clearly, note that it follows from Equations (15) and (18) that, given the  
assumptions of the Bauer-Grossman scheme,

$$\frac{C_{t+1}}{W_{t+1}} = \frac{C_0(1+g^C)^{t+1}}{C_0(1+g^C)^t + (V_0+S_{0,1})(1+g^L)^t} = \frac{1+g^C}{1 + \left( \frac{V_0+S_{0,1}}{C_0} \right) \left( \frac{1+g^L}{1+g^C} \right)^t} \quad (25)$$

Since  $g^L < g^C$ , the last term in the denominator of the right-hand-side expression decays, and  
approaches 0, over time.  $\frac{C_{t+1}}{W_{t+1}}$  therefore increases continually and approaches  $1 + g^C$ . But we

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<sup>10</sup> Equation (24) implies that  $C_t = \left( \frac{\sum \omega_{jt+1} [A_{jt+1}/X_{jt+1}]}{1+g^C} \right) W_{t+1}$ , which means that the constant  
capital-value of time  $t$  ( $C_t$ ) is determined by events that occur at time  $t+1$ . I realize that this  
inverts cause and effect by allowing later events to determine earlier ones. But it is not I who  
allows this. It is a feature/bug of the Bauer-Grossman scheme. Although a variable's growth rate  
is the effect of multiple events that precede the variable's growth, the scheme inverts cause and  
effect by stipulating the magnitudes of its growth rates in advance, prior to time 0. And these  
prespecified growth rates fully determine the variables' future trajectories. Thus, the "time  $t+1$ "  
variables on the right-hand side are able to determine the value of  $C_t$  because they themselves  
have been determined from the start, before time  $t$ . For further discussion of this problem, see my  
forthcoming paper (Kliman 2025).



know from Equation (23) that  $\frac{C_{t+1}}{W_{t+1}} = \sum \omega_{jt+1} \left( \frac{A_{jt+1}}{X_{jt+1}} \right)$ , so  $\sum \omega_{jt+1} \left( \frac{A_{jt+1}}{X_{jt+1}} \right)$  also increases continually and approaches  $1 + g^C$ . As it does so, the net-product share of the total value of output,  $\frac{W_{t+1}-C_t}{W_{t+1}} = 1 - \frac{C_t}{W_{t+1}} = 1 - \frac{\sum \omega_{jt+1} \left( \frac{A_{jt+1}}{X_{jt+1}} \right)}{1+g^C}$ , decreases continually and approaches 0.

### E. The Falling Rate of Profit of the Bauer-Grossman Scheme

In the Bauer-Grossman scheme, the rate of profit is

$$r_{t,t+1} = \frac{S_{t,t+1}}{C_t + V_t} \quad (26)$$

It follows that  $1 + r_{t,t+1}$  equals the ratio of the total value of output to total (constant and variable) capital:

$$1 + r_{t,t+1} = 1 + \frac{S_{t,t+1}}{C_t + V_t} = \frac{C_t + V_t + S_{t,t+1}}{C_t + V_t} = \frac{W_{t+1}}{C_t + V_t} \quad (27)$$

$1 + r_{t,t+1}$  must therefore be less than or equal to the ratio of the total value of output to *constant* capital:

$$1 + r_{t,t+1} \leq \frac{W_{t+1}}{C_t} \quad (28)$$

If there are no discontinued products, we know from Equation (22) that  $\frac{C_t}{W_{t+1}} = \sum \omega_{jt} \left( \frac{A_{jt}}{X_{jt+1}} \right)$ . Hence,

$$1 + r_{t,t+1} \leq \frac{1}{\sum \omega_{jt} \left( \frac{A_{jt}}{X_{jt+1}} \right)} \quad (29)$$

We have also seen that, on average, the physical net product disappears. That is, the denominator of the right-hand side of Inequality (29), the weighted average of the physical “capital-output ratios,” continually increases and approaches 1 (see subsection D, above). As it does so, the right-hand side expression continually falls and approaches 1. So, eventually, as a result of the disappearing average net product,  $1 + r_{t,t+1}$  must be less than or equal to 1, which means that the Bauer-Grossman scheme’s rate of profit must be less than or equal to 0.

If there is some discontinued product, however, there is no physical “capital-output ratio” associated with it, so the fact that the fall in the Bauer-Grossman scheme’s rate of profit is physically determined must be established in a different way. One way is to employ Equation

(24),  $\frac{C_t}{W_{t+1}} = \frac{\sum \omega_{jt+1} \left( \frac{A_{jt+1}}{X_{jt+1}} \right)}{1+g^C}$ . Since this is the reciprocal of  $\frac{W_{t+1}}{C_t}$ , and we know from Inequality (28) that  $1 + r_{t,t+1}$  must be less than or equal to  $\frac{W_{t+1}}{C_t}$ , it follows that

$$1 + r_{t,t+1} \leq \frac{1+g^C}{\sum \omega_{jt+1} \left( \frac{A_{jt+1}}{X_{jt+1}} \right)} \quad (30)$$

The numerator and denominator of the right-hand side ratio are both physically determined, as we have seen. Furthermore, we have seen that it follows from the assumptions of the Bauer-Grossman scheme that the denominator, the weighted average share of physical output accumulated as means of production, increases continually and approaches  $1 + g^C$ , as a result of the physically-determined disappearance of the aggregate value of the net product (see Equation (23)). Thus, the right-hand side expression in Inequality (30) falls continually and approaches  $\frac{1+g^C}{1+g^C} = 1$ . So  $1 + r_{t,t+1}$  must eventually be less than or equal to 1, which means that the rate of profit must be less than or equal to 0.

## Appendix: Growth Rates of Value Magnitudes

### A. Determinants of Growth Rates

The annual growth rate of any variable is the change in its value between one year and the next, divided by its starting-year value. For example, the growth rate of constant capital-value is

$$g_{t,t+1}^C = \frac{C_{t+1} - C_t}{C_t} \quad (A1)$$

and, since  $C_t = \sum p_{jt} A_{jt}$ ,

$$g_{t,t+1}^C = \frac{\sum p_{jt+1} A_{jt+1} - \sum p_{jt} A_{jt}}{\sum p_{jt} A_{jt}} \quad (A2)$$

Thus, in general, the growth rate of constant capital-value (or any other value magnitude) is determined by two factors: changes in prices and changes in the associated physical quantities.

However, the Bauer-Grossman reproduction scheme holds prices constant over time. At the start of his discussion of the scheme, Grossman (2022, p. 110, emphases in original) explicitly stated that “the assumption of *constant* prices, as the *simplest* case, is the one most appropriate for theoretical purposes.” Given this assumption, the growth rates of the scheme’s value magnitudes no longer have two determinants, but only one: changes in the physical quantities. In other

words, the growth rates of the value magnitudes are now *physically determined*. If, for example, constant capital-value increases by 10% per year, it is because, and only because, physical means of production increase by 10% per year. And if the growth rate of means of production rises to 12% or falls to 8%, then so does the growth rate of constant capital-value.

This is fairly obvious. But formalization will be helpful because the precise meaning of “physically-determined rate” is less obvious in the multisector case, where there are multiple means of production, multiple physical products, and so on. I will show that, owing to the constant-price assumption, the growth rate of constant capital-value in the Bauer-Grossman model is the *weighted average of the growth rates of the various physical means of production*. And simply by substituting physical products (outputs) for means of production in the derivations below, one can ascertain that the growth rate of the total value of output in the Bauer-Grossman scheme is the *weighted average of the growth rates of the various physical products*.

## B. Growth Rates in the Bauer-Grossman Scheme (1): the General Multisector Case

Since prices are assumed to be constant, we can omit the prices’ time subscripts. Thus, the growth rate of constant capital-value between times  $t$  and  $t+1$  is

$$\begin{aligned}
 g_{t,t+1}^C &= \frac{\sum p_j A_{jt+1} - \sum p_j A_{jt}}{\sum p_j A_{jt}} = \frac{\sum p A_{t+1}}{\sum p_j A_{jt}} - 1 \\
 &= \frac{p_1 A_{1t+1} + p_2 A_{2t+1} + \dots + p_n A_{nt+1}}{\sum p_j A_{jt}} - 1 \\
 &= \left( \frac{p_1 A_{1t}}{\sum p_j A_{jt}} \right) \left( \frac{p_1 A_{1t+1}}{p_1 A_{1t}} \right) + \dots + \left( \frac{p_n A_{nt}}{\sum p_j A_{jt}} \right) \left( \frac{p_n A_{nt+1}}{p_n A_{nt}} \right) - 1 \\
 &= \left( \frac{p_1 A_{1t}}{\sum p_j A_{jt}} \right) \left( \frac{A_{1t+1}}{A_{1t}} \right) + \dots + \left( \frac{p_n A_{nt}}{\sum p_j A_{jt}} \right) \left( \frac{A_{nt+1}}{A_{nt}} \right) - 1
 \end{aligned} \tag{A3}$$

Since  $g_{t,t+1}^{Aj} = \frac{A_{jt+1} - A_{jt}}{A_{jt}}$  is the (physical) growth rate of the  $j$ -th means of production and the weight of the  $j$ -th means of production is  $\omega_{jt} = \frac{p_j A_{jt}}{\sum p_j A_{jt}}$ —spending on  $A_j$  as a share of total expenditures on means of production at time  $t$ —we can restate Equation (A3) as

$$\begin{aligned}
 g_{t,t+1}^C &= \omega_{1t}(1 + g_{t,t+1}^{A1}) + \dots + \omega_{nt}(1 + g_{t,t+1}^{An}) - 1 \\
 &= \sum \omega_{jt} + \sum \omega_{jt} g_{t,t+1}^{Aj} - 1
 \end{aligned} \tag{A4}$$

And since the sum of the weights is  $\sum \omega_{jt} = \frac{\sum p_j A_{jt}}{\sum p_j A_{jt}} = 1$ , we obtain, finally,

$$g_{t,t+1}^C = \sum \omega_{jt} g_{t,t+1}^{Aj} \quad (\text{A5})$$

which tells us that the (aggregate) growth rate of constant capital, a *value* magnitude, equals the weighted average of the growth rates of the *physical* means of production. (This equality is a consequence of the Bauer-Grossman scheme's assumption that prices remain constant; it does not hold true in general.)

### C. Growth Rates in the Bauer-Grossman Scheme (2): the Multisector Case with New Means of Production

There is, however, one wrinkle to iron out here. The derivation above does not apply if there is some new means of production (an item that serves as a means of production at time  $t+1$  that did not serve as a means of production at time  $t$ ). If some  $A_j$  is new at time  $t+1$ , then  $A_{jt} = p_j A_{jt} = 0$ , so  $A_{jt}$  and  $p_j A_{jt}$  cannot appear in the denominators of ratios used in the derivation above.

What we can properly state, however, is the same result expressed somewhat differently. We know that—if there are no new means of production—then

$$g_{t,t+1}^{Aj} = \frac{A_{jt+1} - A_{jt}}{A_{jt}}, \quad \omega_{jt} = \frac{p_j A_{jt}}{\sum p_j A_{jt}}, \quad \text{and} \quad g_{t,t+1}^C = \sum \omega_{jt} g_{t,t+1}^{Aj}$$

(see Equations (A4) and (A5)). So the growth rate of constant capital-value can be expressed as

$$g_{t,t+1}^C = \sum \left( \frac{p_j A_{jt}}{\sum p_j A_{jt}} \right) \left( \frac{A_{jt+1} - A_{jt}}{A_{jt}} \right) \quad (\text{A6})$$

When some  $A_{jt} = 0$ , however, the right-hand side expression of Equation (A6) is undefined. But (A6) can be rewritten in a manner that eliminates division by 0 and therefore remains valid:<sup>11</sup>

$$g_{t,t+1}^C = \sum \left( \frac{p_j}{\sum p_j A_{jt}} \right) (A_{jt+1} - A_{jt}) \quad (\text{A7})$$

The meaning of the  $\frac{p_j}{\sum p_j A_{jt}}$  term is not intuitively obvious. But if we divide its numerator and denominator by  $\sum p_j$ , we obtain a somewhat more intuitive growth-rate formula:

$$g_{t,t+1}^C = \frac{\sum \left( \frac{p_j}{\sum p_j} \right) (A_{jt+1} - A_{jt})}{\frac{\sum p_j A_{jt}}{\sum p_j}} = \frac{\sum \left( \frac{p_j}{\sum p_j} \right) (A_{jt+1} - A_{jt})}{\sum \left( \frac{p_j}{\sum p_j} \right) A_{jt}} \quad (\text{A8})$$

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<sup>11</sup> Although the term  $\sum p_j A_{jt}$  appears in the denominator of Equation (A7), and we are assuming here that one or more of the  $p_j A_{jt}$  equals 0, we are not dividing by 0. Since at least one of the remaining terms is positive (and none are negative),  $\sum p_j A_{jt}$  itself must be positive.

The numerator of the right-hand-side expression is the weighted average change in the physical quantities of means of production employed. The denominator is the weighted average of the amounts of physical means of production employed at time  $t$ . Because prices remain constant over time, so do the weights, the  $\frac{p_j}{\sum p_j}$ . Thus, when there are new means of production, it continues to be true that, in the Bauer-Grossman scheme, the growth rate of constant capital-value is physically determined. Variations in this growth rate are due solely to variations in the physical quantities of means of production, not to (nonexistent) variations in prices.

It may seem that the weights  $\left(\frac{p_j}{\sum p_j}\right)$  are meaningless, since adding together the prices of different things (i.e., computing  $\sum p_j$ ) is not a valid operation. The sum of \$0.25 per kilowatt hour of electric power and \$8.99 per dozen eggs is meaningless. But the weights are not meaningless if we interpret each numerator as the *cost of one unit* of means of production  $j$  and the denominator as the total *cost of one unit* of each means of production. Interpreted in this manner, the weights are pure (dimensionless) numbers that tell us the fractional contributions of each means of production to this total cost.

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